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MODELING AND TESTING OF THE MUSICAL INSTRUMENT STRING MOTION

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Abstract

This paper considers testing of the musical instrument string motion. First, a mathematical model for an idealized string motion was presented. Simulation results of string tension, mass per unit length and string length influence on a fundamental tone of string vibration were compared to measurement results performed on Fender Stratocaster electric guitar. Furthermore amplitude distribution of string motion considering position of plucking is analyzed. Simulation results of amplitude distribution are performed using Fourier series with taking into account plucking position as an initial condition. These results are compared with frequency spectra obtained from measurements of two different types of guitar i.e. acoustic guitar with steel strings and classic guitar with plastic string.

Key words: stringed musical instruments, plucking, vibrations, acoustics, experiments

1. INTRODUCTION

Stringed musical instruments are musical instruments that produce sound from vibrating strings. They are some of the most represented musical instruments nowadays. In most stringed instruments, the vibrations are transmitted to the body of the instrument, which also vibrates, along with the air inside it. Although in this group belongs many instruments the most common stringed musical instruments are violin and guitar.

Deeper understanding of musical instruments involves theoretical models of their behavior. This knowledge and experience are very important in the field of producing new instruments as well as in the field of restoration of older, used instruments. The physical behavior of a stringed instruments are usually examined under three headings: first the behavior of the stretched string whose vibration is controlled by the player, second the response of the wooden sound box of the instrument and the neighboring area, and the third the radiation of the sound, almost entirely from the sound box. There are many papers and books considering the acoustical measurements on instrument bodies at various stages of its constructions, trying to find the secret of quality sound [1-3].

Although the earlier three mentioned directions of researching of stringed instruments are usually appeared together, in this preliminary paper only the behaviour of the vibrational string and its connection to the produced sound is investigated. The reason for this is that the influence of plucking position as an initial condition, wanted to be explored further as well as that, the relatively cheap measuring equipment, used for the student diploma works, was available.

2. PRODUCING OF THE TONE BY THE STRING INSTRUMENTS

By plucking on the wire its oscillation by the fundamental frequency is excited, but also the frequencies of the higher harmonics can be excited too. Thus a complex tone as a result of the

interference of waves with fundamental tone and any associated higher harmonics, is produced. Thus incurred resulting tones apart by frequency vary by shape, which determines the color tone [4].
On that way the wire instrument produces a very weak sound, but the vibrations are transmitted to the sound box of the instrument, which also has its own frequency, and when these frequencies coincide with the frequencies of vibrating strings, there is a resonance which increase the amplitude tones, or increase the volume of produced tones. The shape of the resulting tone depends on the type of instrument, or the design and performance, so each instrument produces a particular color tone, which is inherent for this type of instrument.

3. MATHEMATICAL MODEL OF IDEALIZED STRING MOTION

In this part of a paper the fundamental relations regarding to the vibrations of the string of the musical instrument will be established and prepared for the use in further chapters.

3.1. Velocity of the wave propagation through the string and its oscillating frequency

Through the string characterized by the linear mass density $\boldsymbol{\mu}$[kgm^{-1}] and tensioned by the force $\boldsymbol{F}$ [N], which is acted on one end by the transversal impulse with the velocity $\boldsymbol{u}$ [ms^{-1}], the disturbance will travel by the velocity

$$v = \sqrt{\frac{F}{\mu}}, \ \text{ms}^{-1} \tag{1}$$

The frequency of oscillation of the wire i.e. the amount of produced tons depends on a tension F [N], the length of the wire L [m], and its weight i.e. the linear density of the wire μ [kgm^{-1}]. For each primary wavelength n=1 it is possible to get a range of higher harmonics n=2,3,… whose frequencies are

$$f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}}, \ \text{Hz} \tag{2}$$

3.2. Vibrational equation of the string

The equation which describes the vibrations of the string of the stringed instruments is known as a one-dimensional wave equation

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \tag{3}$$

To find the solution to this wave equation it is possible to employ the method of separation of variables using the boundary and initial conditions. Boundary conditions include fixed string ends

$$y(0,t) = 0, \quad \text{for each } t > 0, \qquad y(L,t) = 0, \quad \text{for each } t > 0 \tag{4}$$

which gives the solution for the string displacement $y(x,t)$

$$y(x,t) = \sum_{k=1}^{\infty} \sin\left(\frac{k\pi}{L}x\right)\left(\alpha_k \cos\left(\frac{vk\pi}{L}t\right) + \beta_k \sin\left(\frac{vk\pi}{L}t\right)\right) \tag{5}$$

The unknown coefficients α_k and β_k are obtained based on the initial conditions which are created in the moment of the string plucking. In the moment $t = 0$ the string is plucked on the position d and is displaced from the equilibrium position for the distance h. If the $f(x)$ is a function which defines the shape of the string at the moment $t = 0$, the initial conditions are therefore

$$y(x,0) = f(x), \quad \text{for } 0 < x < L \quad \text{and} \quad \frac{\partial}{\partial t} y(x,0) = 0, \quad \text{for } 0 < x < L \tag{6}$$

That give the final solution for the string oscillation of the musical instrument in the form [5]

$$y(x,t) = \sum_{k=1}^{\infty} \frac{2hL^2}{\pi^2 k^2 d(L-d)} \sin\left(\frac{k\pi d}{L} x\right) \sin\left(\frac{k\pi x}{L} x\right) \cos\left(\frac{vk\pi}{L} t\right) \tag{7}$$

4. MEASUREMENTS OF SOME INFLUENCIES ON THE VIBRATIONAL FREQUENCY

To practically demonstrate the influence of the tension, length and weight of the vibration frequency of the wire, several measurements are performed. The electric guitar Fender Stratocaster, a dynamometer to pull the string, precision scale, and a guitar tuner for reading frequency are used.

4.1. Influence of tensioning force on the frequency of the string

The linear mass density of the string used in this experiment was μ=0,0097 [kgm^{-1}] and effective length of the wire was L_{ef}=0.646 [m]. Three measurements were performed and the string frequencies for the different tensioning forces F [N] are shown in the Tab. 1.

Table 1. String frequencies for different tensioning forces

Tensioning force F	50[N]	60[N]	70[N]	80[N]
1. measurement $f_1[Hz]$	176.3	193.8	206.8	223.3
2. measurement $f_2[Hz]$	177.4	191.4	205.1	220.2
3. measurement $f_3[Hz]$	176.1	190.6	208.1	222.8
$f_{av} = \frac{f_1+f_2+f_3}{3}$ [Hz]	**176.6**	**191.9**	**206.7**	**222.1**
Theoretical value $f_{theor} = \frac{1}{2L_{ef}}\sqrt{\frac{F}{\mu}}$ [Hz]	**175.7**	**192.5**	**207.9**	**222.3**

4.2. Influence of the string linear mass density on the frequency of the string

Table 2. String frequencies for different line mass densities

Line mass density μ	0.00097 $[\frac{kg}{m}]$	0.00193 $[\frac{kg}{m}]$	0.003564 $[\frac{kg}{m}]$
1. measurement $f_1[Hz]$	193.8	135.4	101.3
2. measurement $f_2[Hz]$	191.4	134.7	98.8
3. measurement $f_3[Hz]$	190.6	134.9	99.2
$f_{av} = \frac{f_1+f_2+f_3}{3}$ [Hz]	**191.9**	**135.0**	**99.8**
Theoretical value: $f_{theor} = \frac{1}{2L_{ef}}\sqrt{\frac{F}{\mu}}$ [Hz]	**192.5**	**136.5**	**100.4**

String tensioning force in this experiment was F=60 [N] and its effective lenght was L_{ef}=0.646 [m]. Three measurements were performed and densities for different line mass densities μ [kgm^{-1}] are shown in Tab. 2.

4.3. Influence of effective lenght of the string on the frequency of the string

String tensioning force in this experiment was F=60 [N] and the linear mass density was μ=0.0097 [kgm^{-1}]. By plucking of the wire on difference positions (without plucking, plucking on the 5th field, 12th field and 17th field) the effective length of the string L_{ef} has changed, and the frequencies shown in the Tab. 3 were obtained.

Table 3. String frequencies for different effective length of the string obtained by plucking

Effective length of the string L_{ef}	0.646 [m]	0.484 [m]	0.323 [m]	0.240 [m]
Measured f [Hz]	192.4	257.6	385.8	519.3
Theoretical value: $f_{theor} = \frac{1}{2L_{ef}}\sqrt{\frac{F}{\mu}}$ [Hz]	192.5	256.9	385.0	518.1

5. MEASURING OF THE INFUENCE OF PLUCKING POSITION ON THE AMPLITUDES OF THE STRING HARMONICS

By expression (7) the vibration displacement of the string, in relationship to the plucking position, is defined. For the string defined by following parameters:

- length of the string L=1 [m]
- initial displacement of the string obtained by plucking h=0.01 [m]
- tensioning force F=160 [N]
- linear mass density of the string μ=0.004 [kg/m]
- frequency of the string $f = 100$ [Hz],

it is possible to obtain the amplitude diagrams for the different plucking positions. In the case of plucking on the middle of the string (fig. 1), the whole amplitude of vibrations is shown by dashed line and can be obtained by summing of individual contributions of considered harmonics.

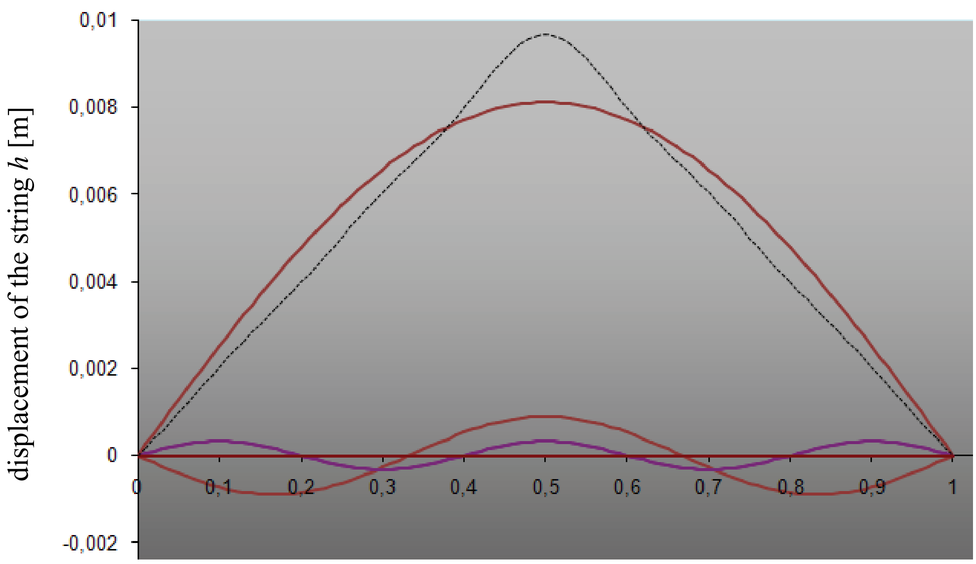

Figure 1: Amplitude diagram for the plucking at position d=L/2

Calculated amplitudes of first 6 harmonics for that case are shown on fig. 2.

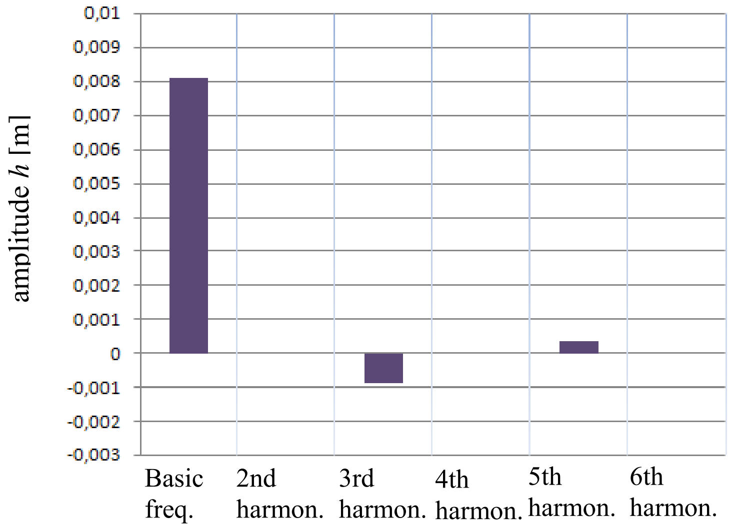

Figure 2. Calculated amplitudes of first 6 harmonics for plucking at position $d=L/2$

Plucking on other place, especially where there is not positioned a vibration node, will give the significant amount for all of the mentioned components, fig. 3.

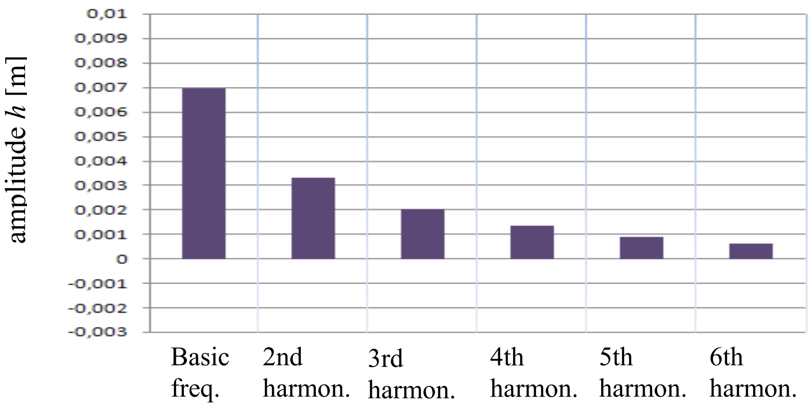

Figure 3. Calculated amplitudes of first 6 harmonics for plucking at position $d=L/10$ from the left end of the string

5.1. Frequency domain digrams

In order to verify theoretically obtained diagrams, a few tones are recorded on the guitar at various positions of plucking. The acoustic guitar with metal strings (E ~ 200 [GPa]) 'Yamaha', type 'FGX-412c', and classical guitar with plastic strings (E ~ 5 [GPa]), 'Rooster'. Sound measurements were done by microphone 'Shure Beta 58A', and for processing and displaying of the tone spectra, the free of charge program 'Spectraciser (Audio Spectrum Analyzer)' was used, (fig. 4.). Program Spectraciser allows to do audio signal analysis using computer's sound card. It can do real-time analysis of a "live" signal fed to computer. It can also do static analysis of an Audio Wave file.

Figure 4. Picture of the guitar and used measuring equipment in the experiment

All tones are played on the 5th string of the guitar (tone A2 – 110Hz). Effective lenght of the wire on both guitars was $L_{ef} = 650[mm]$. In the case of plucking at position *d*=*L*/2, the diagrams on the fig. 5 and 6. are obtained.

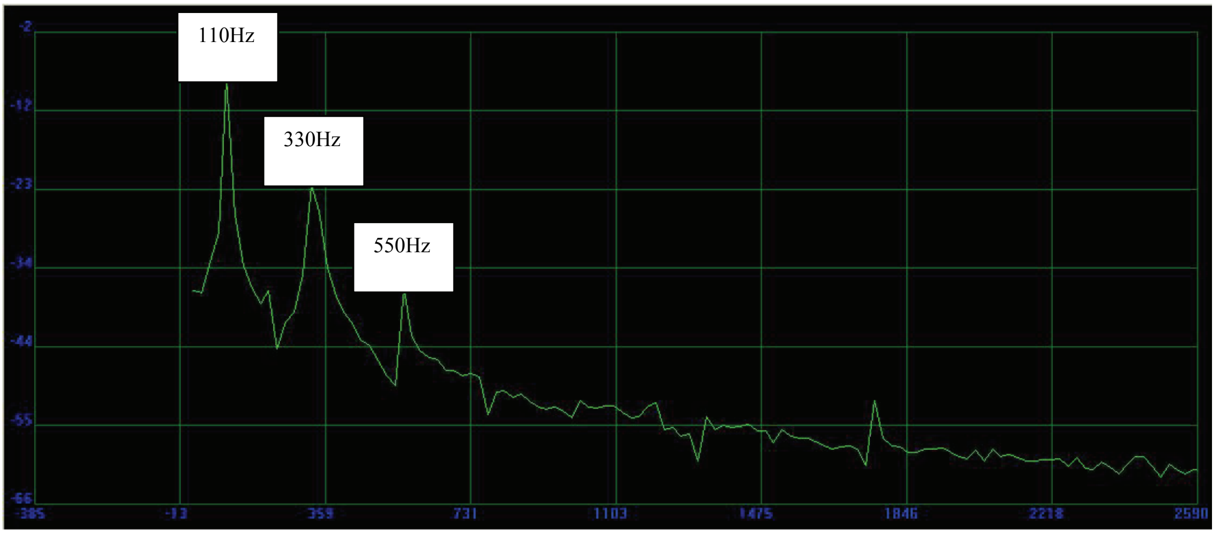

Figure 5. Measured frequency spectrum for plucking at position *d*=*L*/2 (plastic string)

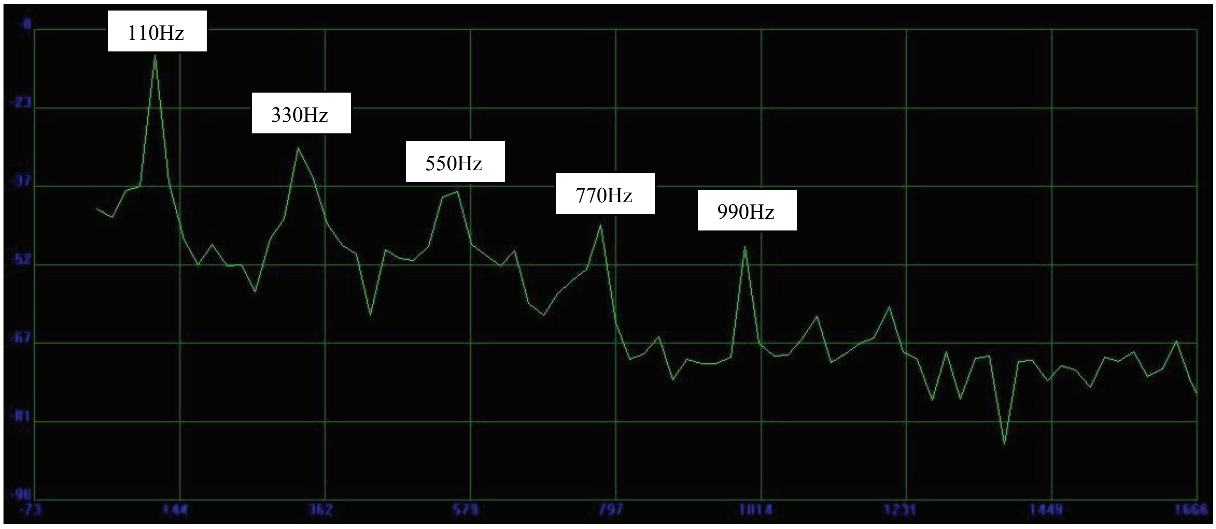

Figure 6. Measured frequency spectrum for plucking at position *d*=*L*/2 (metal string)

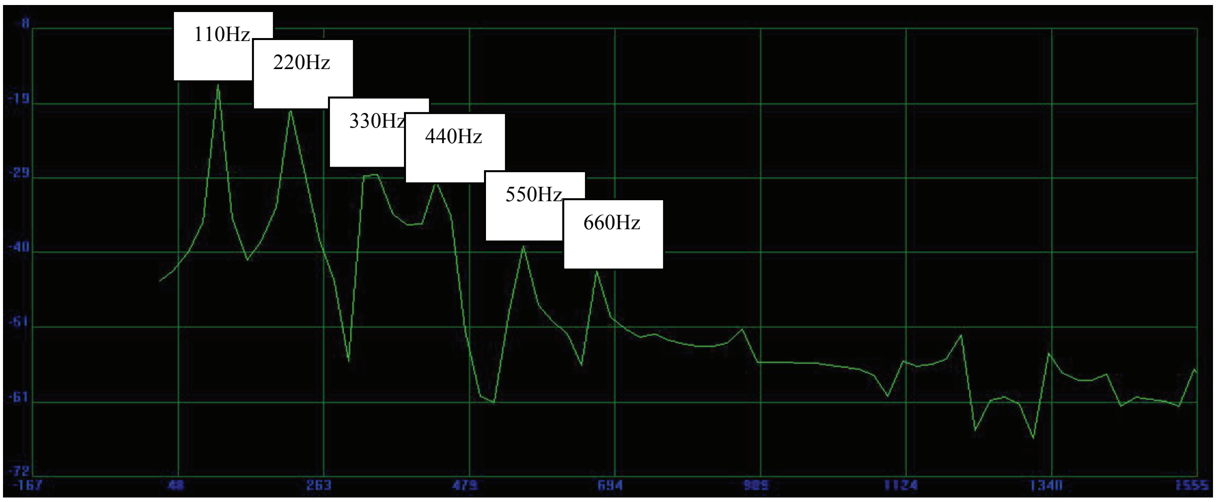

Figure 7. Measured frequency spectrum for plucking at position *d*=*L*/10 (plastic string)

The present results clearly show good agreement in the frequencies and amplitudes of the fundamental frequency (110Hz), and the 3rd and 5th higher harmonics, while the 2nd and 4th and 6th hamonic don´t appear, which corresponds to the theoretical model. Good coincidence measurements with a mathematical model is obtained in the case of plucking at position *d*=*L*/10 from the left end of the string. These results are shown at the diagrams on the fig. 7 and 8.

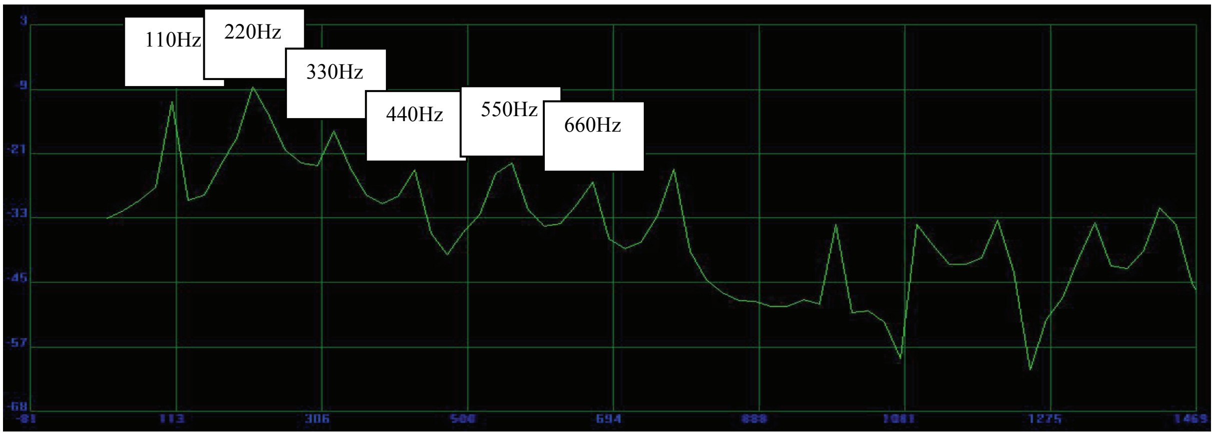

Figure 8. Measured frequency spectrum for plucking at position *d*=*L*/10 (metal string)

Analyzing the frequencies of recorded tones, obtained by plucking in different positions of the guitar strings, and comparing the results with theoretical diagrams, one might concluded that the results matched in greater part, which confirm theoretical model of the vibrating strings.

One could expect that not all the results fully coincide because the amplitude of certain frequencies of recorded tones does not depend only on the vibration of the strings but also on the structure of the instrument i.e. on its own frequencies, so each instrument amplify some frequencies more and some less. Smaller mistakes are also possible due to the small deviations from the correct position *d* during the plucking of the string, and also on the frequency spectrum of the recorded sound affect specifications of the microphone.

5.2. Measured spectrograms

Spectograms recorder during the experiments (plastic and metal wires) are given on the fig. 9 and 10.

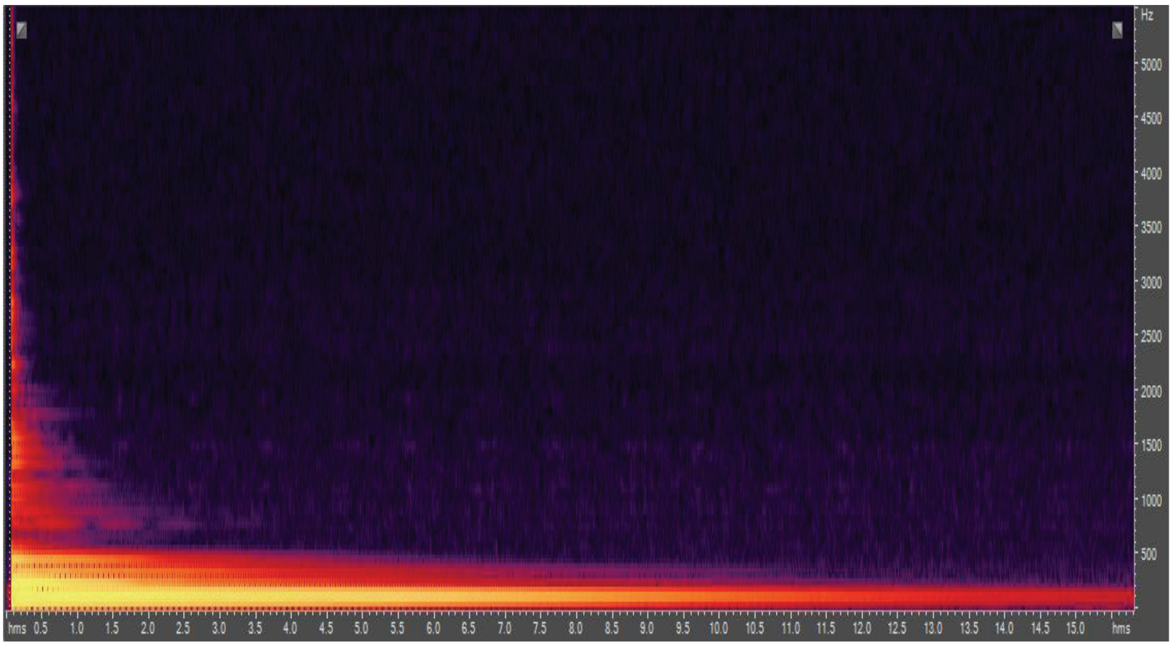

Figure 9. Spectograme of the tone A2 (110 Hz) played on the classical guitar (plastic string)

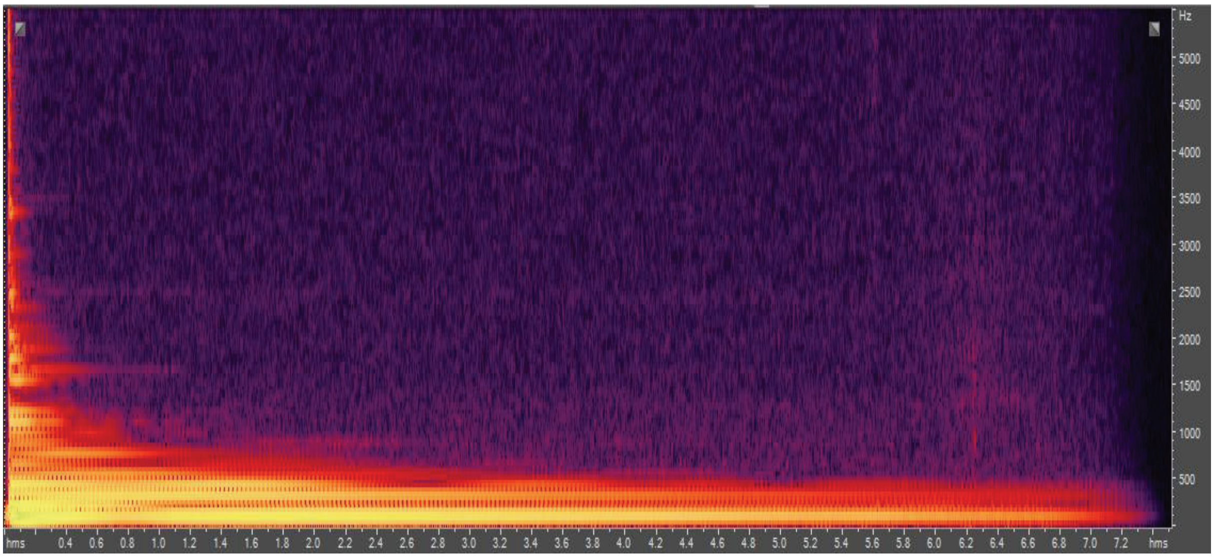

Figure 10. Spectograme of the tone A2 (110 [Hz]) played on the acoustic guitar (metal string)

Recorded spectograms show a high degree of similarity in the case of using classical (plastic string) and acoustical (metal string) guitar. In both cases there is evident frequency content (higher harmonics) above the 3000 [Hz].
When this spectograms should be compared with piano spectograms, the difference would be drastic. For the same tone A2 (110Hz), on acoustic guitar one might obtain a much wider range of frequencies than on the piano, or according higher frequencies are more pronounced in the case of guitar, while frequencies above 3000Hz on piano practically can´t be heard. On the other hand one might note that the lower harmonics on the piano have more balanced intensity than on the acoustic guitar, which also affects the color of the resulting tone of an instrument.

6. CONCLUSIONS

The paper presents the first results of vibration and acoustical tests on the musical instruments with strings. The experimental tests mainly confirmed the results from mathematical model. Minor deviations between mathematical simulation and measurement results were noticed. The reason for this lies within the facts that simulation model is idealized while in the experimental model, natural frequencies depend on music instrument structure as well and due to slight deviation of the exact plucking position during the measurements from position taken in simulation.

7. ACKNOWLEDGMENTS

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REFERENCES

[1] E. Ravina, “Violins characterization through vibro-acoustic experiments”, Proceedings of the Acoustics 2012 Nantes Conference 23-27 April 2012, Nantes, France.

[2] M. French, G. Bissinger, “Testing of Acoustic Stringed Musical Instuments – An Introduction”, Experimental Techniques, Vol.25., No.1., 40-43 (2001.)

[3] M. Campbell, C. Greated, “The *musician's guide to acoustics*”, Oxford University Press, 2001.

[4] T.D. Rossing, “Handbook of acoustics”, Springer, (2007)

[5] A.W. Leissa, M.S.Qatu, “Vibrations of continuos systems”, Mc-Graw Hill, 2011.

[6] …, “Spectraciser (Audio Spectrum Analyzer) - manual”, available on http://www.fountainware.com/Products/Spectracizer/index.htm